

I Pentru fiecare raspuns corect se acorda 5p  $\Rightarrow 5 \times 10 = 50$

- ① D, ② A ③ C ④ C ⑤ C ⑥ D  
 ⑦ B ⑧ B ⑨ B ⑩ C

II a)  $\frac{2^{x+y}-3}{2^x+5} = \frac{2y+1}{3y+1} = \frac{z^2+1}{3z+1} \cdot \frac{2y+1}{3y+1} < 1 \dots \dots \dots 3p$

$\Rightarrow \frac{z^2+1}{3z+1} < 1 \Rightarrow z^2+1 < 3z+1 \Rightarrow z^2 < 3z \Rightarrow z < 3 \dots \dots 3p$

pt  $z=1 \Rightarrow \frac{2y+1}{3y+1} = \frac{1}{2} \Rightarrow 4y+2=3y+1 \Rightarrow y=-1 \notin \mathbb{N} \dots \dots 2p$

pt  $z=2 \Rightarrow \frac{2y+1}{3y+1} = \frac{5}{7} \Rightarrow 14y+7=15y+5 \Rightarrow y=2 \dots \dots 2p$

$\Rightarrow \frac{2^{x+2}-3}{2^x+5} = \frac{5}{7} \Rightarrow 28 \cdot 2^x - 21 = 5 \cdot 2^x + 25 \dots \dots 2p$

$\Rightarrow 23 \cdot 2^x = 46 \Rightarrow 2^x = 2 \Rightarrow x=1 \dots \dots 1p$

$(x, y, z) = (1, 2, 2) \dots \dots \dots \boxed{15p}$

b)  $x^2 = 5y^2 + 2012 \Rightarrow u(5y^2 + 2012) \in \{2, 7\}$

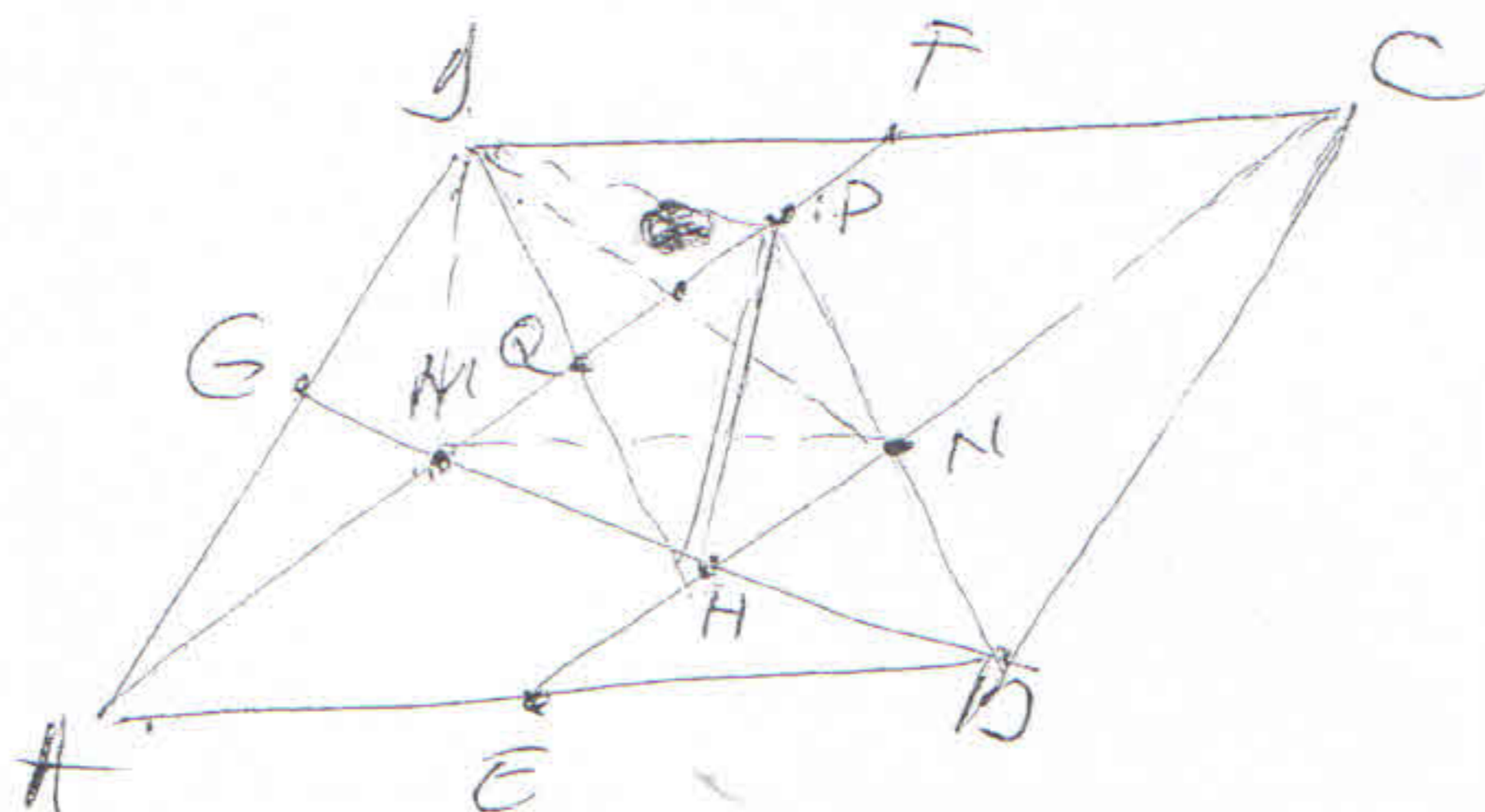
$\Rightarrow u(x^2) \in \{2, 7\}$  fals  $\Rightarrow$

nu exista  $x, y \in \mathbb{N}$

5p

$\boxed{5p}$





Fig

$AF \cap BN = \{P\}$

$DN \cap AF = \{Q\}$

$CF \parallel AE \Rightarrow AECT \Rightarrow \text{parallelogram} \Rightarrow AF \parallel CE$  3p

$\Delta ABP$ , E = midpoint of  $AB$ ,  $EN$  = line through E  $\Rightarrow N$  = midpoint of  $BP$  2p  
 $EN \parallel AF$

$\Delta HMB$ ,  $EH \parallel MB$   $\Rightarrow EH$  = line through E  $\Rightarrow H$  = midpoint of  $BM$  2p  
 E = midpoint of  $AB$

$\Delta MBP$ ,  $HQ \parallel BP$   $\Rightarrow HQ$  = line through H  $\Rightarrow Q$  = midpoint of  $MP$  2p  
 H = midpoint of  $BM$

$\Delta HCB$ ,  $QF \parallel HC$   $\Rightarrow QF$  = line through Q  $\Rightarrow Q$  = midpoint of  $HP$  2p  
 F = midpoint of  $BC$

$Q$  = midpoint of  $HP$  and  $Q$  = midpoint of  $MP$   $\Rightarrow \Delta MHP$  parallelogram 3p

$\Rightarrow DP \parallel MH$   
 $\Delta ADP$ ,  $GM \parallel DP$   $\Rightarrow GM$  = line through G  $\Rightarrow M$  = midpoint of  $AP$  2p  
 G = midpoint of  $AD$

$N$  = midpoint of  $BP$  and  $M$  = midpoint of  $AP$   $\Rightarrow$  ~~MN~~  
 $\Rightarrow MN$  = line through midpoints in  $\Delta PAB$  2p

$\Rightarrow MN \parallel AB$  2p

20p